Government Engineering College, Nawada

Department of Applied Science & Humanities (Mathematics)

Assignment Sheet-I

Session	**: 2019-20(Even Sem.)**	**Semester**	**: II**
Course/ Branch	**: B. Tech./EE**	**Paper Name & Code**	**: Mathematics-II (103202)**
Module	**: 1**	**Topic Covered**	**: Matrices**

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Note: Following are the problems which are required to be done by the students for an overall understanding of the topics.

1. Prove that every square matrix can be written as sum of symmetric and skew symmetric matrices.
2. Prove that every square matrix can be written as sum of Hermitian and skew Hermitian matrices.
3. Express given matrix A as sum of a symmetric and skew symmetric matrices
$$A=\begin{bmatrix}6 & 8 & 5\\ 4 & 2 & 3\\ 1 & 7 & 1\end{bmatrix}.$$
4. Express the following matrix as the sum of a Hermitian and skew-Hermitian matrices.
$$\begin{bmatrix}2+3i & 0 & 3i\\ 5 & i & 7\\ 1-i & -1+i & 8\end{bmatrix}$$
5. Prove that $\frac{1}{2}\begin{bmatrix}1+i & -1+i\\ 1+i & 1-i\end{bmatrix}$ is a unitary matrix.
6. Find the matrix P which diagonalizes the matrix $A=\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$. Varify that $P^{-1}AP=D$, where D is the diagonal matrix.
7. Using elementary transformations, find the inverse of the given matrix
(i) $A=\begin{bmatrix}1 & 2 & 1\\ 3 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}$ **(ii)** $A=\begin{bmatrix}2 & 1 & -1\\ 0 & 2 & 1\\ 5 & 2 & -3\end{bmatrix}$ **(iii)** $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$
8. Find the rank of the matrix by reducing it to Echelon form (triangular form)
$$A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}.$$
9. Find the rank of the following matrix by reducing it to normal form (canonical form).
(i) $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$ **(ii)** $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 7\\ 3 & 6 & 10\end{bmatrix}$ **(iii)** $A=\begin{bmatrix}1 & 2 & -1 & 3\\ 4 & 1 & 2 & 1\\ 3 & -1 & 1 & 2\\ 1 & 2 & 0 & 1\end{bmatrix}$.
10. Determine the non-singular matrices P and Q such that PAQ is in the normal form for A.
$$A=\begin{bmatrix}2 & 1 & -3 & -6\\ 3 & -3 & 1 & 2\\ 1 & 1 & 1 & 2\end{bmatrix}$$. Hence find the rank of A.

11. Check the consistency of the following system of equation and if consistent then find the solution :

(i) $\begin{array}{l} 2x-y+3z-9=0 \\ x+y+z-6=0 \\ x-y+z-2=0 \\ x+y-z=0 \end{array}$ **(ii)** $\begin{array}{l} x+y+z=6 \\ 2x+3y-2z=6. \\ 5x+y+2z=13 \end{array}$

12. Determine the values of a and b for which the system $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+az=b$ has **(i)** no solution, **(ii)** unique solution and **(iii)** infinitely many solutions

13. Investigate for what values of λ and μ the simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have **(i)** no solution, **(ii)** unique solution and **(iii)** infinitely many solutions.

14. Investigate for what values of λ and μ the simultaneous equations $x+2y+z=6$, $x+4y+3z=10$, $x+4y+\lambda z=\mu$ have **(i)** no solution, **(ii)** unique solution and **(iii)** infinitely many solutions.

15. If the system of equations $x+ay+az=0, bx+y+bz=0, cx+cy+z=0$, where a, b, c are non-zero and non-unity, has a nontrivial solution, show that $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=-1$.

16. For what values of k the equations $x+y+z=1$, $2x+y+4z=k$, $4x+y+10z=k^2$ have a solution and solve them completely in each case.

17. Find the values of θ for which the system of linear equations $2(sin\theta)x+y-2z=0, 3x+2(cos\theta)y++3z=0, 5x+3y-z=0$ has a non-trivial solution.

18. Show that the following system have no solutions unless $a+b+c=0$, in which case they have infinitely many solutions, find their solutions $a=1, b=1$ and $c=-2$, $-2x+y+z=a$, $x-2y+z=b.,x+y-2z=c$.

19. Using Caley-Hamilton theorem to find the inverse of the matrix **(i)** $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ **(iv)** $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **(v)** $A=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ and hence find A^{-1}.

20. Show that the matrix $A=\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ satisfies the equation $A(A-I)(A+2I)=0$.

21. Determine the eigenvalues and eigenvectors of the smallest eigen value of the following matrix :

(i) $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$.

22. Prove that all the eigenvalues of a Hermitian matrix are real.

23. Prove that the modulus of each eigenvalue of a unitary matrix is unity.

24. Show that the matrix $A=\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. Then, obtain the matrix $B=A^2+5A+3I$.

25. Examine whether the matrix A, where A is given by (i) $A=\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (ii) $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

26. A square matrix $A=\begin{bmatrix}1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$ .find the modal matrix P and the resulting diagonal matrix D of A.

27. Reduce the matrix $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$ into a diagonal matrix.

28. Find the matrix A whose eigenvalues and the corresponding eigenvectors are as given in following (i) Eigenvalues: 2,2,4; Eigenvectors: $(-2,1,0)^T,(-1,0,1)^T,(1,0,1)^T$ (ii) Eigenvalues: 1,2,3; Eigenvectors: $(1,2,1)^T,(2,3,4)^T,(1,4,9)^T$

29. Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form.

30. Find index and signature of the quadratic form $x^2+2y^2-3z^2$.

31. Find the nature of quadratic form $x^2+5y^2+z^2+2yz+6zx+2xy$.

Text / Reference Books:

1. Peter V. O' Neil, A text book of Engineering Mathematics, Thomson (Cengage Learning), 2nd Edition, 2010.
2. B.S.Grewal, Advanced Engineering Mathematics, Khanna Publishars, 40th Edition, 2010.
3. E. Kreyszig, "Advanced Engineering Mathematics", John Wiley and Sons, New York, 2005.
4. B.V. Ramanna, "Higher Engineering Mathematics", Tata Mcgraw Hill Publishing Company Ltd., 2008.
5. R.K. Jain and S.R.K. Iyengar, "Advanced Engineering Mathematics", Narosa Publishing House, 2008.

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(Signature of the Faculty Member with date)