Government Engineering College, Nawada

Department of Applied Science & Humanities (Mathematics)

Assignment Sheet-II

Session	**: 2019-20(Even Sem.)**	**Semester**	**: II**
Course/ Branch	**: B. Tech./ CE**	**Paper Name**	**: Mathematics-II (101202)**
Module	**: 3B**	**Topic Covered**	**: ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDERS**

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Note: Following are the problems which are required to be done by the students for an overall understanding of the topics.

1. Examine whether the following functions are linearly independent (i) $1, \cos x, \sin x$ (ii) $\ln x, \ln x^2, \ln x^3$ (iii) $e^{-x}, \sin x, \cos x$ (iv) x, x^2, x^3 (v) e^x, e^{2x}, e^{3x}.
2. Find a general solution of the following differential equations: (i) $y''-4y = 0$ (ii) $y''-y'-2y = 0$ (iii) $y'' + y'-2y = 0$ (iv) $y''-4y'-12y = 0$ (v) $y'' + 9y' = 0$ (vi) $9y''-12y' + 4y = 0$ (vii) $y''-y'-6y = 0$ (viii) $4y'' + 4y' + y = 0$ (ix) $y'''-y''-5y' + 6y = 0$ (x) $8y'''-12y'' + 6y'-y = 0$ (xi) $y^{iv}-a^2y = 0$ (xii) $y^{iv} + 32y'' + 256y = 0$.
3. Solve the following differential equations:

   (i) $y''' + y = e^x + 2e^{-x}$ (ii) $y''-4y' + 3y = \sin 3x \cos 2x$ (iii) $y''-4y' + 4y = e^x + \sin 2x$ (iv) $y'' + 4y = \cos x \cos 3x$ (v) $y''-4y = x^2$ (vi) $y''-2y' + 3y = \cos x + x^2$ (vii) $y''-4y' + 4y = x^2 + e^x + \cos 2x$ (viii) $y''-2y' + y = xe^x \sin x$ (ix) $y''-3y' + 2y = \sin 2x + xe^x$.
4. Solve by method of variation of parameters: (i) $y'' + a^2y = \sec ax$ (ii) $x^2y'' + xy'-y = x^2e^x$ (iii) $x^2y''-4xy' + 6y = \sin(\log x)$ (iv) $y'' + 4y = \cos x$ (v) $y'' + a^2y = \operatorname{cosec} ax$ (vi) $y'' + y = \tan x$ (vii) $y'' + 6y' + 9y = \frac{e^{-3x}}{x}$ (viii) $y'' + 4y' + 4y = e^{-2x}\sin x$ (ix) $y''-y = \frac{2}{1+e^x}$ (x) $x^2y'' + xy'-y = x^2y$ (xi) $x^2y'' + 3xy' + y = \frac{1}{(1-x)^2}$.
5. Solve the following differential equations: (i) $xy''-(2x-1)y' + (x-1)y = 0$ (ii) $(1-x^2)y'' + xy'-y = x(1-x^2)^{3/2}$ (iii) $y''-\cot x\, y'-(1-\cot x)\, y = e^x \sin x$.
6. Solve: (i) $y''-2\tan x\, y' + y = 0$ (ii) $y''-4xy' + (4x^2-1)y = -3e^{x^2}\sin 2x$ (iii) $x^2y''-2(x + x^2)y' + (x^2 + 2x + 2)y = 0$ (iv) $(\cos x)\, y'' + (\sin x)\, y'-2(\cos^3 x)y = 2\cos^5 x$.
7. Solve: (i) $x^3y''' + 2x^2y'' + 2y = x + \frac{1}{x}$ (ii) $x^2y''-5xy' + 3y = \ln x$ (iii) $(x + 1)^2y'' + (x + 1)y' + y = 4\cos(\log(1 + x))$ (iv) $(2x + 5)^2y'' + 6(2x + 5)y' + 8y = x$.
8. Find the regular and singular points of the differential equations (i) $(1-x^2)y''-2xy' + n(n + 1)y = 0$ (ii) $x^2y'' + axy' + by = 0$.
9. Classify the singular points of the following equations (i) $x^2y'' + (\sin x)y' + (\cos x)y = 0$ (ii) $x^3(x^2-1)y''-x(x + 1)y'-(x-1)y = 0$ (iii) $x^2y'' + 2xy' + (x^2-n^2)y = 0$.
10. Find the power series solution about $x = 0$, of the differential equation (i) $y''-2y = 0$ (ii) $(1-x^2)y''-2xy' + 2y = 0$.

11. Find the power series solution about x = 2 of the equation $y'' + (x-1)y' + y = 0$.

12. Find the series solutions of the following differential equations by the Frobenious method: (i)) $x^2y'' + 2xy' + (x^2-n^2)y = 0$ (ii) $9x(1 + x)y''-6y' + 2y = 0$ (iii) $(1-x^2)y''-2xy' + 6y = 0$.

13. Find the series solutions about the indicated point of the following differential equations by the Frobenious method: (i) $2(1-x)y''-xy' + y = 0$, x=1 (ii) $x(x-2)y'' + 4y' + 3y = 0, x = 2$.

14. Express $P(x) = 3P_3(x) + 2P_2(x) + 4P_1(x) + 5P_0(x)$ as polynomial in x, where $P_m(x)$ is the Legendre polynomial of order m.

15. Express $f(x) = x^4 + 2x^3-6x^2 + 5x-3$ in the terms of Legendre polynomials.

16. Show that (i) $P_n(1) = 1$ (ii) $P_n(-x) = (-1)^n) P_n(x)$ (iii) $P'_n(1) = n(n + 1)/2$ (iv) $\int_{-1}^{1} P_n(x)\,dx = 0$ (v)) $\int_{-1}^{1} P_n^2(x)\,dx = \frac{2}{2n+1}$.

17. Show that $J_n(x)$ is a even function for n even and an odd function for n odd where n is an integer.

18. Prove that (i) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ (ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ (iii) $J_3(x) = \left(\frac{8}{x^2}-1\right)J_1(x)-\frac{4}{x}J_0(x)$ (iv) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{1}{x^2}(3-x^2)\sin x-\frac{3}{x}\cos x\right]$ (v) $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{1}{x^2}(3-x^2)\cos x + \frac{3}{x}\sin x\right]$.

Text / Reference Books:

1. Peter V. O` Neil, A text book of Engineering Mathematics, Thomson (Cengage Learning), 2nd Edition, 2010.
2. B.S.Grewal, Advanced Engineering Mathematics, Khanna Publishars, 40th Edition, 2010.
3. E. Kreyszig, "Advanced Engineering Mathematics", John Wiley and Sons, New York, 2005.
4. B.V. Ramanna, "Higher Engineering Mathematics", Tata Mcgraw Hill Publishing Company Ltd., 2008.
5. R.K. Jain and S.R.K. Iyengar, "Advanced Engineering Mathematics", Narosa Publishing House, 2008.

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19/08/2020

(Signature of the Faculty Member with date)