Government Engineering College, Nawada

Department of Applied Science & Humanities (Mathematics)

Tutorial Sheet-VII

Session	**: 2019-20(Even Sem.)**	**Semester**	**: II**
Course/ Branch	**: B. Tech./ CE**	**Paper Name**	**: Mathematics-II (101202)**
Module	**: 5A**	**Topic Covered**	**: Solution of Algebraic & Transcendental Equation**

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Note: Following are the problems which are required to be done by the students for an overall understanding of the topics.

Questions based on bisection method

1. Solve the equation $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$ by the method of bisection.
2. Solve the equation $x^4 + 2x^3 - x - 1 = 0$ in the interval (0,1) by the method of bisection.
3. Find a positive real root of $x - cos\, x = 0$ by bisection method, correct up to four decimal places between 0 and 1.
4. Find an approximate value of the root of the equation $3x - \sqrt{1 + sin\, x} = 0$ by bisection method.
5. Find the real root of $x\, log_{10} x = 1.2$ by bisection method, correct up to four decimal places.
6. Find a real root of $x^3 - x = 1$ between 1 and 2 by bisection method.

Questions based on false position (Regula-Falsi) method

1. Solve: $x^3 - 5x + 3 = 0$ by using Regula-Falsi method.
2. Find the real root of the equation $xe^x = cos\, x$ in the interval (0,1) by using Regula-Falsi method correct to four decimal places.
3. Apply False position method to find smallest positive root of the equation $x - e^{-x} = 0$, correct to the three decimal places.
4. Find the real root of the equation $3x + sin\, x - e^x = 0$ by the method of False position, correct up to four decimal places.
5. Find the real root of the equation $x\, log_{10} x - 1.2 = 0$ correct to five decimal places.

Questions based on Newton Raphson method

1. Find a real root of the equation $x = e^{-x}$ using the Newton - Raphson method.
2. Use Newton-Raphson method to find the smallest positive root of the equation $tan\, x = x$.
3. Explain the method of Newton–Raphson for computing roots. Apply it for finding x from $x^2 - 25 = 0$.
4. Explain the order of convergence and prove that Newton–Raphson method is second order convergent.
5. Find the value of $\sqrt{65}$ by using the Newton-Raphson method.
6. If $u_x$is a function of x for which fifth difference are constant and $u_1 + u_7 = -786,\ u_2 + u_6 = 686$,$u_3 + u_5 = 1088$ Find u_x.
7. Derive Newton-Raphson method to find a root of the equation f(x) = 0. Prove that this method has quadratic convergence.
8. Show that the equation $f(x) = cos\left\{\frac{\pi(x+1)}{8}\right\} + 0.148x - 0.9062 = 0$ has one root in the interval $(-1,0)$ and one in $(0,1)$. Calculate the negative root correct to 4 decimal places by Newton-Raphson method.

Miscellaneous Questions

1. For what values ofa, b, and c the order of the iterative method $x_{n+1} = ax_n + b\frac{N}{x_n} + c\frac{N^2}{x_n^3}$ for computing $\sqrt{N}$ becomes as high as possible. For this choice of a, b and c find the value of asymptotic error constant.
2. How should the constant α be chosen to ensure the fastest possible convergence with the iteration formula $x_{n+1} = \frac{\alpha x_n + {x_n}^{-2} + 1}{\alpha + 1}$.
3. Show that the following two sequences both have convergence of the second order with same limit $\sqrt{a}$.

$$x_{n+1} = \frac{1}{2}x_n\left(1 + \frac{a}{{x_n}^2}\right) and\ x_{n+1} = \frac{1}{2}x_n\left(3 - \frac{{x_n}^2}{a}\right).$$

Text / Reference Books:

1. R. K. Jain & S. R. K. Iyengar. "Advanced Engineering Mathematics," Narosa Publishing House Pvt. Ltd., 3 Ed., 2011.
2. P. Kandasamy, K. Thilagavathy, K. Gunavathi, Numerical Methods, S. Chand & Company, 2nd Edition, Reprint 2012.

3. S.S. Sastry, Introductory Methods of Numerical Analysis, Phi, 4th Edition, 2005.
4. Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
5. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 35th Edition, 2010.